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The 27-plet baryons with spin 3/2 under SU(3) symmetry

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Abstract. We investigate the spin-3/2 baryons in the 27-plet based on flavor $SU(3)$ symmetry. For $J^p = 3/2^+$, we find all the candidates for non-exotic members. For $J^p = 3/2^-$, we predict a new non-exotic member $\Lambda(1780)$. Fitting the mass spectrum and calculating the widths of the members show an approximate symmetry of the 27-plet of $SU(3)$. We find that the exotic members have relatively large widths and the $\Xi(1950)$ has spin and parity $J^p = 3/2^-$. The possibility of assigning the non-exotic candidates to an octet is also analyzed.

PACS. 11.30.Hv Flavor symmetries – 12.39.Mk Glueball and nonstandard multi-quark/gluon states – 13.30.Eg Hadron decays

1 Introduction

The $SU(3)$ classification scheme proposed by Gell-Mann and Ne'eman in 1961 has been proved quite successful and fruitful in the investigation of hadron spectroscopy. In this classification scheme, one can group the experimentally known strongly interacting particles with the same quantum numbers of spin and parity into various irreducible representations of the $SU(3)$ group. There are several $SU(3)$ multiplets which have been well established by this means, for example, $J^P = \frac{1}{2}^+$ octet and $J^P = \frac{3}{2}^+$ decuplet baryons, which supplied clear and unambiguous evidence for the $SU(3)$ classification scheme. Higher multiplets are also allowed in $SU(3)$, such as $\bar{10}$, 27, 35, etc. Because they contain the so-called exotic states beyond the three-quark qqq content in the language of the conventional quark model as Gell-Mann mentioned [1] and could hardly be found in early year's experiments, the higher-multiplet scheme received little attention. From quantum chromodynamics (QCD), the underlying theory of the strong interaction, the possibility for the existences of exotic states cannot be ruled out. The chiral soliton model (χSM) [2] motivated investigations on the antidecuplet which contains the exotic state Θ^+ reported first by LEPS Collaboration later [3]. The Θ^+ state has the minimal quark content $uudd\bar{s}$ [4] and hence is an exotic pentaquark state with positive strangeness. The higher multiplet 27, which contains an isovector Θ, also attracted some attention. χSM predicted a new isotriplet of Θ baryon [5–7], with its mass being about 1.6 GeV and width about 80 MeV. With the flux-tube quark model and the QCD sum rules, Kanada-En'yo *et al.* predicted the Θ with $I(J^p) = 1(3/2^-)$ and mass 1.4–1.6 GeV [8, 9]. Noticeably, recently the STAR Collaboration at RHIC presented data indicating a small but significant Θ^{++} candidate with mass of about 1528 MeV [10]. Though there are more and more negative reports against the existence of the $\Theta^+(1540)$ at present, it is worthwhile to explore these new exotic particles.

In this paper, we examine possible non-exotic candidates of the 27-plet with spin 3/2 in the baryon particle listings from the Particle Data Group [11] by calculating their masses and partial decay widths based on the approximate flavor $SU(3)$ symmetry of the strong interaction. Up to the present, seldom works about the exotics tried to approach this issue using the most general and model-independent method $SU(3)$. In the quark model, these non-exotic candidates were often assigned to the 56-plet of $SU(6)$ with orbital excitation. However, in ref. [6], it could also get a rather good result from the chiral soliton model without demanding such analysis. This means that we can try a more general analysis. Our motivation is an attempt to verify whether the $SU(3)$ symmetry, which has been greatly successful in hadron physics [12, 13], can continue to play an important role in the investigation of new particles. By this means, Guzey and Polyakov have reviewed the spectrum of all baryons with mass less than approximately 2000–2200 MeV and catalogued them into twenty-one $SU(3)$ multiplet including 1, 8, 10 and $\bar{10}$ [14]. That work can be viewed as an update

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of ref. [13]. Likewise, the present study does not depend on any specific model, neither does it introduce any presupposed adjustable parameters. So, it is rather general and can be compared with results of other works which are model dependent.

2 Mass spectrum and decay width

The main tools of $SU(3)$ systematization are the well-known Gell-Mann–Okubo (GMO) mass formulae and the calculations of two-body hadronic decays as showed in refs. [12–14]. We will follow this classical treatment process. Firstly, the masses of baryons in the 27-plet can be obtained by using the GMO mass formula:

$$M = M_0 + \alpha Y + \beta D_3^3, \tag{1}$$

where M_0 is a common mass of a given multiplet and $D_3^3 = I(I+1) - Y^2/4 - C/6$ with $C = 2(p+q) + \frac{2}{3}(p^2+pq+q^2)$ for the (p,q) irreducible representation. α and β are mass constants that depend on the representation which the baryon belongs to. For the 27-plet, (p,q) is $(2,2)$, whose weight diagram and the labels for the member states are shown in fig. 1. Then, we can get

$$\begin{aligned}
\Theta_1 &= M_0 + 2\alpha - \frac{5}{3}\beta, & \Delta_{27} &= M_0 + \alpha + \frac{5}{6}\beta,\\
N_{27} &= M_0 + \alpha - \frac{13}{6}\beta, & \Sigma_{27} &= M_0 - \frac{2}{3}\beta,\\
\Lambda_{27} &= M_0 - \frac{8}{3}\beta, & \Xi_{27} &= M_0 - \alpha - \frac{13}{6}\beta,\\
\Sigma_{27,2} &= M_0 + \frac{10}{3}\beta, & \Xi_{27,3/2} &= M_0 - \alpha + \frac{5}{6}\beta,\\
\Omega_{27,1} &= M_0 - 2\alpha - \frac{5}{3}\beta. & &
\end{aligned} \tag{2}$$

From above, one can find some interesting relations, such as the similar octet GMO relation

$$2N_{27} + \Xi_{27} = 3\Lambda_{27} + \Sigma_{27}, \tag{3}$$

and five independent equal-spacing rules

$$\begin{aligned}
\Theta_1 - \Delta_{27} &= \Delta_{27} - \Sigma_{27,2},\\
\Sigma_{27,2} - \Xi_{27,3/2} &= \Xi_{27,3/2} - \Omega_{27,1},\\
\Theta_1 - N_{27} &= N_{27} - \Lambda_{27},\\
\Lambda_{27} - \Xi_{27} &= \Xi_{27} - \Omega_{27,1},\\
N_{27} - \Xi_{27} &= \Delta_{27} - \Xi_{27,3/2}.
\end{aligned} \tag{4}$$

In order to determine the mass spectrum, one just needs to know the masses of three certain states in the 27-plet and requires that they do not satisfy eq. (4) at the same time. For the case of $J^p = 3/2^+$, we choose the following three well-established resonances taken from PDG as inputs: $\Delta(1600)$, $N(1720)$ and $\Lambda(1890)$. For the case of $J^p = 3/2^-$, we choose $\Delta(1940)$, $N(1720)$ and $\Sigma(1940)$. Other inputs are possible, but give no more candidates in PDG than themselves. The best mass fitting results are shown in table 1. Note that we now have two sets of 27-plet baryons. For the set of $J^p = 3/2^+$, all the non-exotic

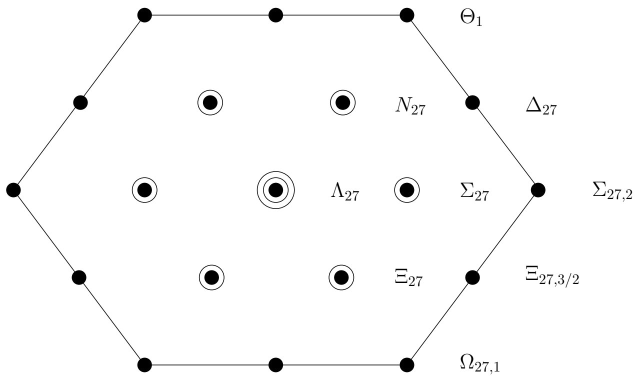

Fig. 1. Weight diagram for the 27-plet baryons.

states have their candidates. For the set of $J^p = 3/2^-$, we get a new state Λ_{27} with mass 1780 MeV. The existence of a new Λ hyperon with $J^p = 3/2^-$ was predicted in specific constituent quark models with various assumptions about the quark dynamics [15,16]. Reference [15] gave a mass of about 1780 MeV and model A in ref. [16] gave a mass of 1775 MeV. Here, we directly get it only by completing the $SU(3)$ picture of the 27-plet baryons. This result comes from model-independent analysis, which just shows the merit of $SU(3)$. In both sets, all exotic states have no candidates at present.

Next, we will calculate the two-body partial widths of the 27-plet baryons decaying to the octet baryons and the pseudoscalar mesons to verify this assignment. The $SU(3)$ invariant 27-8-8 interaction Lagrangian can be obtained by constructing the $SU(3)$ singlet like this form

$$L = g_{27}\bar{T}_{ij}^{kl} B_k^i M_l^j, \tag{5}$$

where T_{ij}^{kl} is an irreducible tensor notation to represent the 27-plet baryons, and B_k^i and M_l^j denote the baryon octet and pseudoscalar meson octet, respectively. The full expression written in terms of the physical states has been deduced in [17]. For the concrete decay process of a 27-plet baryon B' with spin 3/2 to an octet baryon B with spin 1/2 and a pseudoscalar meson M

$$B' \to B + M, \tag{6}$$

the calculation can be performed in the framework of Rarita-Schwinger formalism. The parity-conserving interaction Lagrangian are [18,19]

$$\begin{aligned}
L_+ &= \frac{g_{B'BM}}{m_\pi}\bar{\psi}\Psi^\mu\partial_\mu\phi,\\
L_- &= i\frac{g_{B'BM}}{m_\pi}\bar{\psi}\gamma_5\Psi^\mu\partial_\mu\phi,
\end{aligned} \tag{7}$$

for B' with positive- or negative-parity respectively, where ψ is the spin-1/2 field and Ψ^μ is the spin-3/2 field. The pseudoscalar meson field is ϕ and the factor $1/m_\pi$ is introduced to make the coupling constant $g_{B'BM}$ relative to the universal coupling constant g_{27} in eq. (5) dimensionless. We can get the coupling constant $g_{B'BM}$ by directly computing the Clebsch-Gordan coefficient among the $SU(3)$

Table 1. The masses and widths of baryons in the 27-plet (in units of MeV).

	Candidate	Width in PDG	Decay mode	Branching ratio	Γ_i(exp)	Γ_i(th)	Γ_i(th) $[g_{27}^2]$
$J^p = 3/2^+$							
$\Delta_{27}(1600)$	$\Delta(1600)$	250–450	$N\pi$	10%–25%	25–112.5	129	3579.3
$N_{27}(1720)$	$N(1720)$	100–200	$N\pi$	10%–20%	10–40	10(input)	277.5
			$N\eta$	$(4.0 \pm 1.0)\%$	3–10	30.9	857.1
			ΛK	1%–15%	1–30	10.5	291.7
			ΣK			0.2	5.1
$\Sigma_{27}(1810)$	$\Sigma(1840)$	120 ± 10	$N\bar{K}$	0.37 ± 0.13	26.4–65	46.5	1290.4
$\Lambda_{27}(1890)$	$\Lambda(1890)$	60–200	$N\bar{K}$	20%–35%	12–70	14	388.2
			$\Sigma\pi$	3%–10%	1.8–20	2.3	63.8
$\Xi_{27}(2020)$	$\Xi(2030)$	20^{+15}_{-5}	$\Lambda\bar{K}$	$\sim 20\%$	3–7	85.9	2384.8
			$\Sigma\bar{K}$	$\sim 80\%$	12–28	7.3	202.9
$\Theta_1(1550)$	?	?	NK	?	?	33.2	920.1
$\Sigma_{27,2}(1650)$	?	?	$\Sigma\pi$	?	?	164.4	4562.1
$\Xi_{27,3/2}(1900)$	?	?	$\Xi\pi$	?	?	125.8	3491.2
			$\Sigma\bar{K}$	?	?	3.4	95.5
$\Omega_{27,1}(2150)$	?	?	$\Xi\bar{K}$	?	?	232.9	6461.8
$J^p = 3/2^-$							
$\Delta_{27}(1940)$	$\Delta(1940)$	460 ± 320	$N\pi$	0.18 ± 0.12	8.4–234	136.9	1155.2
			ΣK			9.7	81.7
$N_{27}(1700)$	$N(1700)$	50–150	$N\pi$	5%–15%	2.5–22.5	2.5(input)	21.1
			$N\eta$	$(0.0 \pm 1.0)\%$	0–1.5	3.7	31.3
			ΛK	$< 3\%$	< 4.5	0.3	2.7
			ΣK			0.0002	0.002
$\Sigma_{27}(1940)$	$\Sigma(1940)$	150–300	$N\bar{K}$	$< 20\%$	< 60	20.5	172.6
$\Lambda_{27}(1780)$	?	?	$N\bar{K}$	?	?	0.3	16.2
			$\Sigma\pi$	?	?	0.18	1.5
$\Xi_{27}(1940)$	$\Xi(1950)$	60 ± 20	$\Lambda\bar{K}$	seen		9.9	83.8
			$\Sigma\bar{K}$	possibly seen		0.5	4.5
			$\Xi\pi$	seen		0.9	7.7
$\Theta_1(1620)$	?	?	NK	?	?	48.9	412.3
$\Sigma_{27,2}(2260)$	?	?	$\Sigma\pi$	?	?	400	3375.9
$\Xi_{27,3/2}(2180)$	?	?	$\Xi\pi$	?	?	63.8	538.5
			$\Sigma\bar{K}$	?	?	56.4	475.6
$\Omega_{27,1}(2100)$	?	?	$\Xi\bar{K}$	?	?	19	160

irreducible representations of B', B, M. Accordingly, the decay widths are written as

$$\Gamma_+(B' \to BM) = \frac{g^2_{B'Bm}}{12\pi m_\pi^2} p^3 \frac{\left[(m_{B'} + m_B)^2 - m^2\right]}{m^2_{B'}},$$
$$\Gamma_-(B' \to BM) = \frac{g^2_{B'Bm}}{3\pi m_\pi^2} p^5 \frac{1}{\left[(m_{B'} + m_B)^2 - m^2\right]}, \tag{8}$$

where p is the c.m. momentum value of the final meson. In terms of the baryons masses $m_{B'}$, m_B and the meson mass m, we have

$$p = \frac{\sqrt{\left[(m_{B'} + m_B)^2 - m^2\right]\left[(m_{B'} - m_B)^2 - m^2\right]}}{2m_{B'}}. \tag{9}$$

After some trivial calculations, we get the results expressed by the universal coupling constant g_{27}^2 and list them particularly in table 1. To examine whether the $SU(3)$ symmetry can hold, we need to compare all the ratios of two certain partial decay widths in table 1 with the data from experiment. Here, we choose the minimum experimental value of $\Gamma(N_{27} \to N\pi)$ as input just to show the validity. Similarly, other choices can easily be verified. We also list the results in table 1.

3 Discussion

Up to now, we only considered the pure 27-plet assignment. Actually, the 27-plet can mix with other representations, such as the octet or the decuplet with spin 3/2 equally. Because the well-established $J^p = 3/2^+$ decuplet works well for the mass spectrum, this will imply that the mixing with the 27-plet is small. Possibly, mixing can take place between the set of the 27-plet with $J^p = 3/2^-$ and the potentially octet with the same quantum numbers in the available particle listings [20]. But from ref. [14], the possible pure octet assignment of $N(1520)$, $\Lambda(1690)$, $\Sigma(1670)$ and $\Xi(1820)$ seems to work well too. So, we do not treat the mixing explicitly here.

Because of the similar octet GMO relation eq. (3), it seems that $N_{27}, \Sigma_{27}, \Lambda_{27}$ and Ξ_{27} can be assigned to a pure octet. We need to calculate their partial decay widths to

Table 2. The widths of baryons in the octet (in units of MeV).

	Candidate	Width in PDG	Decay mode	Branching ratio	Γ_i(exp)	Γ_i(th)	Γ_i(th) $[g_8^2]$
$J^p = 3/2^+$							
$N_{27}(1720)$	$N(1720)$	100–200	$N\pi$	10%–20%	10–40	27.4	$2081.3(d+f)^2$
			$N\eta$	$(4.0 \pm 1.0)\%$	3–10	1.8	$79.4(d-3f)^2$
			ΛK	1%–15%	1–30	1.7	$27(d+3f)^2$
			ΣK			0.01	$38.3(d-f)^2$
$\Sigma_{27}(1810)$	$\Sigma(1840)$	120 ± 10	$N\bar{K}$	0.37 ± 0.13	26.4–65	0.4	$1613(d-f)^2$
$\Lambda_{27}(1890)$	$\Lambda(1890)$	60–200	$N\bar{K}$	20%–35%	12–70	13.2	$215.7(d+3f)^2$
			$\Sigma\pi$	3%–10%	1.8–20	3	$1276d^2$
$\Xi_{27}(2020)$	$\Xi(2030)$	20^{+15}_{-5}	$\Lambda\bar{K}$	$\sim 20\%$	3–7	5(input)	$220.7(d-3f)^2$
			$\Sigma\bar{K}$	$\sim 80\%$	12–28	20(input)	$1521.8(d+f)^2$
$J^p = 3/2^-$							
$N_{27}(1700)$	$N(1700)$	50–150	$N\pi$	5%–15%	2.5–22.5	10(input)	$158.3(d+f)^2$
			$N\eta$	$(0.0 \pm 1.0)\%$	0–1.5	1(input)	$2.9(d-3f)^2$
			ΛK	$< 3\%$	< 4.5	0.1	$0.3(d+3f)^2$
			ΣK			0.001	$0.02(d-f)^2$
$\Sigma_{27}(1940)$	$\Sigma(1940)$	150–300	$N\bar{K}$	$< 20\%$	< 60	6.1	$215.8(d-f)^2$
$\Lambda_{27}(1780)$	?	?	$N\bar{K}$	?	?	4	$9(d+3f)^2$
			$\Sigma\pi$	?	?	0.1	$30d^2$
$\Xi_{27}(1940)$	$\Xi(1950)$	60 ± 20	$\Lambda\bar{K}$	seen		2.7	$7.8(d-3f)^2$
			$\Sigma\bar{K}$	possibly seen		15.2	$33.8(d+f)^2$
			$\Xi\pi$	seen		1.6	$57.8(d-f)^2$

examine this possibility. Again, we construct the $SU(3)$ invariant 8-8-8 interaction Lagrangian by two possible couplings, namely, the well-known f- and d-type interactions. We write the interaction Lagrangian as

$$L = g_8(d+f)\bar{P}_i^l B_k^i M_l^k + g_8(d-f)\bar{P}_i^l B_l^k M_k^i, \qquad (10)$$

where $\bar{P}_i^l$ represents the octet which consists of $N_{27}, \Sigma_{27}, \Lambda_{27}$ and Ξ_{27}, and B_k^i and M_l^k denote the baryon octet and the pseudoscalar meson octet, respectively. After an analogous process as the foregoing, we list the results expressed by the universal coupling constant g_8 and two parameters f, d in table 2. The appropriate fitting results of these partial decay widths are also presented. Compared with the results from the 27-plet, the picture of the octet seems to be able to give right relative magnitudes of the partial decay widths of a certain baryon. For example, $\Gamma(N_{27} \to N\pi)$ should be broader than $\Gamma(N_{27} \to N\eta)$ according to the experimental data while the picture of 27-plet gives the reverse results both in $SU(3)$ and χSM [6]. But, there are still some partial decay widths such as $\Gamma(\Sigma_{27} \to N\bar{K})$ which are depressed too low as can be seen from table 2. Simultaneously, note that the analysis [16] predicts that the Λ couples very weakly to the $N\bar{K}$ state. The calculation from 27-plet supports this to be more reasonable than that from the octet. Although no appropriate f/d ratio can be found to be compatible with all experimental data, the picture of the octet still cannot be completely excluded especially for the case of $J^p = 3/2^-$ because of the large widths and the imprecise branch ratios of its members. More exact experimental data are needed to examine this possibility. Of course, the picture of the 27-plet is more attractive, as it provides also the connection of $\Delta(1600)$ and $\Delta(1940)$ with other 27-plet members with $J^P = 3/2^+$ and $J^P = 3/2^-$ respectively, especially those in the center-of-weight diagram of the 27-plet. So it contains more information than the picture of the octet. Θ^{++} can also be contained in a higher multiplet such as the 35-plet. However, we know that for the 35-plet, any of its members cannot decay to an octet baryon and a pseudoscalar meson because of the confinement coming from the group theory. Actually, many particles in PDG as potential candidates have such large decay branch. Therefore we think that the existence of a multiplet higher than the 27-plet is unlikely possible under the $SU(3)$. For the possible Θ^{++} referred to in refs. [8–10], the 27-plet is very hopeful of containing it.

4 Summary

In summary, we use the flavor $SU(3)$ symmetry to examine the possible candidates of the 27-plet with spin 3/2. By calculating the partial decay widths of the candidates, the approximate symmetry of the 27-plet of $SU(3)$ can be seen. For $J^p = 3/2^-$ multiplet, we predict a new missing baryon $\Lambda(1780)$, no matter in the picture of the 27-plet or octet. The picture of the 27-plet provides also the connection of $\Delta(1600)$ and $\Delta(1940)$ with other members of $J^P = 3/2^+$ and $J^P = 3/2^-$ baryons. Compared with the results from χSM, the non-exotic candidate $\Xi(1950)$ has $J^p = 3/2^-$, which was predicted to be $J^p = 3/2^+$ in ref. [6]. In both cases, one can find that the exotic members have relatively larger widths than those of non-exotic members, which makes them more difficult to be detected experimentally. The results obtained here are model independent, and would be useful for the future study of new baryons by combining with other dynamical approaches.

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